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Summary Sheet

The emergence of sustainable forms of energy, such as wind or solar power, has increased the viability of living 'off the grid' for many homeowners. When removing oneself from traditional power infrastructure is an option, why not go further and live fully independently? However, with solar power, in particular, the issue of energy storage is a stubborn one. Solar power production ceases at night, leaving any renewable energy-powered house stranded until sunrise. To circumvent this issue, many off-the-grid homeowners default to battery storage systems; arrays of batteries that permit the long-term storage of electrical energy, making the hurdle of nighttime null. As an added redundancy, many of these systems are not only capable of powering through the night, but through cloudy days as well. Our task is to generate a model which evaluates a sufficient and realistic battery storage system for any remote house powered fully by solar energy. Our battery system should be capable of meeting nighttime power needs as well as having cloudy day redundancy.

Our model outputs an amount of a specific battery that meets our house's requirements. Any battery can be tiled to be sufficient, but some batteries will do this more efficiently than others. Thus, our output must consist not only of the batteries themselves but also incorporate cost, as cost is the best metric in our case to compare the relative efficacy of different batteries. With this in consideration, our model must be able to compare a variety of batteries on the market and produce an easily-interpretable cost value for each so the superior battery system may be chosen. Our model considers the appliance usage within the home in respect to time and occupants, the constant background energy usage of appliances such as HVAC, and even factors in the exact length of a night. It combines these factors over a comprehensible and easily-expandable platform, and finally utilizes realistic electrical principles to calculate the needed quantity and cost of our considered batteries.

For a base 1600ft^2 house, our model deemed 4 Tesla Powerwall+ batteries, for $34000USD, to be acceptable. Checking this value against readily available battery storage systems, which generally cost ~$35000, our output is right on the nose. Furthermore, our generalized model almost always outputs Tesla's battery as the superior choice, granting solid mathematical insight on the state of the battery market and guidance on the correct battery to choose for solar energy storage. The exact quantity of batteries required is also specified in our output. While the output may be imperfect (and this is difficult to check due to limited online data), we have a strong base structure that could easily be altered to generate extremely accurate results if needed.

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1 – Introduction

1.1 – Background

With the development of sustainable energy sources as alternatives to fossil fuels comes an increasingly important question: How can sustainable energy, such as solar or wind power, be effectively stored? An expanding number of homeowners must solve this dilemma as they begin to utilize sustainable energy sources such as solar panels to power their homes. Currently, the prevailing solution is electrical batteries, which store energy in the form of electricity for later use. There are many different batteries on the market, with a large variation between the design and application of each. Every home has different electrical needs and therefore requires a unique energy storage solution, posing a major challenge to many homeowners seeking to run their homes off sustainable energy. Aiding homeowners in this undertaking is the basis of our task.

1.2 – Restatement of Problem

We must develop a model which determines the best battery storage system for a 1600ft^2 house powered fully by solar panels. Our chosen battery system must support the house through the night as well as through a cloudy day. In generating this model, a multitude of factors must be considered: the total energy needed to support the house over a cloudy day, the ‘flow’ required to run all the appliances in the house, and the differing capabilities of batteries already on the market. The model must then take these considerations and output an adequate power system, likely an array of multiple batteries. Afterward, our model must be adapted to output an adequate battery storage system for any home, and the viability of cement batteries must be explored.

2 – Preliminary Information

2.1 – Assumptions

1. During light hours, our house’s solar array is capable of fully supporting the house’s power needs while also charging the battery storage system to its full capacity.

Justification: The task assumed by our team was only to determine the battery storage system of the house. Our battery system is only required to support the house at night and on a cloudy day, so the solar panels must be able to support the house during the day itself. Furthermore, we must assume that the batteries can be reliably charged to their full capacity during a non-cloudy day since the specifications of the battery system are otherwise negated.

2. No energy is generated during night hours, where night is sunset to sunrise.

Justification: Solar panels produce a negligible electrical output when the sun is not in the sky; for the sake of simplicity, this production will be disregarded. Therefore, the home’s electrical demands must be fully met by the battery storage system during the night.

3. No energy is produced during a cloudy day.

Justification: Calculating the amount of energy produced during a cloudy day would require knowledge of the house's solar grid and its regular output, which is beyond the scope of our task. Cloudy days also vary in severity, meaning that it is not possible to assign a simple coefficient of energy production to every cloudy day. Consequently, our model must default to the assumption of no energy production during a cloudy day. This assumption also gives our system redundancy.

4. The length of a cloudy day is 24 hours plus one night.

Justification: In the event of a cloudy day, the batteries will have to power the house until the solar panels can become productive again. The batteries will have to last the night before the cloudy day and the cloudy day itself, totaling 24 hours plus one night.

5. A day is split into 2 periods, 16 productive hours and 8 rest hours. Only constant power is drawn during rest hours, and rest hours only occur at night.

Justification: The habits of the house's inhabitants will have a direct effect on the power necessities of the house; therefore, it must be assumed that they have a regular period over which they use nonconstant appliances. This regularity allows our model to be concise and simpler to figure.

6. Appliances can be used at any time during productive hours.

Justification: while it is reasonable to assume rigid periods of sleep and productivity, appliance usage is much more variable. For the sake of versatility, we assumed that appliance usage does not follow a strict guideline and can occur at any time throughout the productive period.

7. Power consumption not listed does not exist.

Justification: To calculate every object and device that requires power within a house is extremely complex and variable; to do so would overburden our model. Additionally, unlisted power consumption would be inconsequential to our output.

8. All listed batteries can be combined in series and/or parallel.

Justification: Running batteries in series and/or parallel increases their capacity or continuous power rating respectively, permitting flexibility in our battery system. While some batteries cannot be connected in such a manner, for sake of simplicity in calculation, this is not a limitation our model must consider.

2.2 – Notes and Definitions

Throughout our paper, we repeatedly utilize terms some may be unfamiliar with. For sake of convenience, their definitions can be referenced here:

Capacity: A straightforward measure, the amount of energy required to run our house (kWh). Regarding batteries, it represents the quantity of usable energy they can store (kWh).

CPR: The sustained current or power draw of our house (kW). Regarding batteries, it represents their maximum sustained output adjusted for efficiency (kW).

Series/Parallel Running: An electrical grouping of batteries in which their CPR ratings or capacity ratings are summed respectively.

We also use a variety of reputable sources in generating our model. These will be marked with a flag in the form [x] which refers to a source in the works cited (pages 24-25) under the same flag.

2.3 – Variables

Variable	**Purpose**	**Units**
E_n	The house's energy consumption during the night.	kWh
E_d	The house's energy consumption during light hours.	kWh
E_{Cloudy}	The house's energy consumption over a cloudy day.	kWh
E_{batt}	The capacity rating of a given battery.	kWh
e_{App}	Energy consumption of a given appliance over 24hrs.	kWh
e_{HVAC}	Energy consumption of HVAC over 24hrs.	kWh
W_{App}	The power draw of a given appliance.	kW
W_{HVAC}	The power draw of HVAC	kW
cpr	The maximum power draw used by the house.	kW
cpr_{min}	The base power draw required to run the house.	kW
cpr_{batt}	The CPR rating of a given battery.	kW
p	The number of inhabitants of the house.	People
H_f	The square footage of the house.	ft^2
n	The length of a night.	hrs
ha	The hour angle of the sun.	Radians
A_x	The surface area of individual sections of the house.	ft^2
lat	The latitude of the house's location.	Degrees
t	The internal temperature of the house.	Fahrenheit
c	External temperature.	Fahrenheit
len_{series}	The maximum length of a series grouping.	Batteries
$len_{parallel}$	The length of a parallel grouping.	Batteries
len_{min}	The minimum length of a series grouping.	Batteries
Q_{series}	The quantity of maximum series groupings.	Batteries
$batt_{Tot}$	The total amount of needed batteries.	Batteries
$cost$	The cost of the battery storage system.	USD

Table 2a – A collection of the equation-centric variables used throughout our paper.

3 – Formulating a Model

3.1 – Initial Considerations

The first step in formulating our model was brainstorming what factors we wanted to consider. Currently available batteries have 3 major statistics: Their capacity, their continuous power rating (CPR), and their instantaneous power rating (IPR). Capacity is straightforward, being the quantity of energy (in kWh) that the battery can store, it defines how long the house can be run. CPR is effectively the maximum sustainable current of the battery, it defines how much of the house the battery will be able to run at once. Finally, IPR defines the maximum instantaneous current of the battery, it defines how 'powerful' an appliance can be and still receive adequate power (E.g., a blender). We decided upon capacity and CPR as the most important of the two, as they largely determine the performance of the battery in powering a home. IPR, being by nature only important for small bursts of time, was deemed less important.

3.2 – Research

With these considerations in mind, we may begin preliminary research, focusing on the cornerstone appliances of any home. The table that follows is the fruit of our research, and it will be the basis for appliance calculations in the future:

Appliances	Power Use Per Day:	Power Draw:
Lighting [9]	3.862 kWh/day	0.455 kW
Water Heater [17]	9.674 kWh/day	4.5 kW
Fridge/Freezer Combo [14]	1.932 kWh/day	0.610 kW
HVAC [18]	8.096 kWh/day	1.642 kW
Microwave [10]	0.567 kWh/day	1.700 kW
Laundry Machine [11]	0.433 kWh/day	1.320 kW
Dishwasher [13]	0.655 kWh/day	1.800 kW
Oven [25]	2.6 kWh/day	2.800 kW
Stove [26]	1.267 kWh/day	4.800 kW
TV [12]	0.318 kWh/day	0.106 kW

Table 3a – A collection of the appliances used in our house with energy use and power draw.

Appliance data turned out to be less available than initially assumed, so many of these values had to be approximated or found using yearly averages.

3.3 – Interpretation

With this newfound data, we can begin interpretation. Our appliances first must be split into 2: constant and nonconstant. Our appliances first must be split into 2 periods: constant and nonconstant. Nonconstant appliances are straightforward, drawing a given amount of power only when a user activates them. They are also only used during productive hours, as defined in assumption 5. However, constant appliances operate at all hours, requiring a base amount of power at any time (including sleep hours. This disparity in power usage and timing needed to be considered in our model, and the table following defines how we split our appliances.

Constant Appliances	**Nonconstant Appliances**
Water Heater	Lights
HVAC	Microwave
Fridge/Freezer Combo	Laundry Machine
	Dishwasher
	Oven
	Stove
	TV

Table 3b – The appliances defined in 3a sorted by energy usage type.

3.4.1 – Equations and Calculations: Capacity

With these considerations made, we can begin to define the capacity and *cpr* requirements of our house.

Capacity was the first of the two problems to tackle. While it may be ideal to assume all of our appliances run on a schedule that minimizes power usage per day, this is simply not the case in real life, as explained in assumption 6. Chances are all appliances would eventually need usage on the same day, and our batteries must be prepared for such an event, lest the house be stranded powerless until light. As such, we set e_{App} (the energy usage of nonconstant appliances over 24hrs, in kWh), equal to:

$$e_{app} = \sum \text{Nonconstant Appliance Energy Usage}$$
$$= 9.702\text{kWh}$$

While this does not account for multiple uses of an appliance per day, it is robust enough for our purposes. e_c (the energy usage of constant appliances over 24hrs, in kWh) must be calculated separate due to assumption 5, but it follows the same format:

$$e_c = \sum \text{Constant Appliance Energy Usage}$$
$$= 19.702\text{kWh}$$

With energy usage solved, we can formulate our model. As a reminder, our battery storage system needs to power the house through the night as well as through a cloudy day. Using our definition for a cloudy day from assumption 4, we can produce a basic equation for E_{Cloudy} (the house's energy consumption over a cloudy day):

$$E_{Cloudy} = E_d + 2E_n$$

In solving E_{Cloudy}, we must define its components E_n and E_d (Energy usage of the home over night and day). Both these variables rely heavily on a single variable, n, the length of a night. Due to the changing nature of nighttime on Earth, however, finding a suitable value for n is a very involved process.

n changes according to 2 factors, the time of year and latitude[4], but we can make some concessions to simplify our solution. First, we set the latitude of our home to 45°, an arbitrary, but suitably remote value taken for simplicity's sake. Addressing the time of year requires some extra logic: Solving for n as a function of the year would be a valid approach, but our battery system must operate on a daily cycle. Thus, we only need to solve for a single n, specifically the maximum. It is known that this maximum occurs on the winter solstice (December 21st) for latitudes >0. Thus, our value of n will be solved for a latitude of 45° and on the 21st of December. Using data amassed by the NOAA[3], solving for n with these inputs evaluates to:

$$n = 15.24\text{hrs}$$

With this value for n, the equations for E_n and E_d can both be formulated quite trivially. First, constant power will always be on, so we can easily split its power draw into day and night portions:

$$E_n = \frac{n}{24}(e_c)$$

$$E_d = \frac{24-n}{24}(e_c)$$

Our last step is including e_{App}. By assumptions 5 & 6, e_{App} is relevant during all productive hours, but 'dormant' 8 hours per night. Factoring this into our equations, we can evaluate them as follows:

$$E_n = \frac{n}{24}(e_c) + \frac{n-8}{24}(e_{App}) = \frac{15.24\text{hrs}}{24\text{hrs}}(19.702\text{kWh}) + \frac{15.24\text{hrs} - 8\text{hrs}}{24\text{hrs}}(9.702\text{kWh})$$

$$E_n = 15.43\text{kWh}$$

$$E_d = \frac{24-n}{24}(e_c + e_{App}) = \frac{24\text{hrs} - 15.24\text{hrs}}{24\text{hrs}}(19.702\text{kWh} + 9.702\text{kWh})$$

$$E_d = 10.73\text{kWh}$$

Thus, E_{Cloudy} is equal to:

$$E_{Cloudy} = E_d + 2E_n = 10.73\text{kWh} + 2(15.43\text{kWh})$$

$$E_{Cloudy} = 41.59\text{kWh}$$

This defines the required capacity of the battery storage system for our 1600ft^2 home.

3.4.2 - Equations and Calculations: CPR

With capacity evaluated, we must formulate *cpr*. This calculation follows the same logic as capacity, but we must make a small modification. As we would later come to find, having every appliance on in the house at once results in our output almost completely hinging on *cpr*; capacity has little impact. Therefore, we decided to limit our simultaneous appliance power usage to ½ its full sum, limiting the maximum quantity of appliances turned on at once for sake

of flexibility (except for constant appliances which by nature are always on). With this in mind, our equation for *cpr* (the maximum power draw required to run the house)
came to be:

$$cpr = \frac{\sum \text{Nonconstant Appliance Power Draw}}{2} + \sum \text{Constant Appliance Power Draw}$$

$$= 6.49\text{kW} + 6.752\text{kW}$$

$$= 13.24\text{kW}$$

Since *cpr* is time-independent, constant appliance power draw can be included in this general calculation.

3.5 – Final Evaluations

The following section requires an understanding of 2 basic electrical principles: when batteries are run in parallel, their capacity ratings are summed but their *cpr* rating remains the same. The opposite is true for running in series. With this knowledge, we can generate the formulas which will define the needed quantity of individual batteries for our battery storage system. In the following equations, $len_{parallel}$ represents the number of batteries in a parallel grouping needed to meet capacity, while len_{series} represents the same for series and *cpr*:

$$len_{parallel} = \text{ceil}\left(\frac{E_{Cloudy}}{cap_{batt}}\right) \qquad len_{series} = \text{ceil}\left(\frac{cpr}{cpr_{batt}}\right)$$

Where “ceil” is the ceiling function.

This solution is adequate upon first look, but there is a crippling issue, especially with small batteries. This results in a near-useless battery storage system, as the parallel grouping would have the CPR rating of a single battery while the series grouping would have the capacity rating of a single battery, both of which are unsatisfactory. To rectify these limitations, we must have 2 additional constraints: the number of series groupings needed to sustainably run the house, Q_{series}, and a minimum series grouping size, len_{min}.

Q_{series} requires a quantity of time we want to run at *cpr*, which we have arbitrarily decided to be 1/10th of the day. len_{min} requires a cpr_{min}, which is simply the *cpr* equation with a different constant multiple of nonconstant appliance power draw. This constant we decided upon was 1/8, implying that our home will be able to have at least 1/8th of its appliances on at any given productive time. The following are our solutions for Q_{series} and len_{min}:

$$\frac{(Q_{series})(cap_{batt})}{E_{Cloudy}} \geq \frac{1}{10} \qquad cpr_{min} = \frac{\sum \text{Nonconstant Appliance Power Draw}}{8} + \sum \text{Constant Appliance Power Draw}$$

$$Q_{series} = \text{ceil}\left(\frac{E_{Cloudy}}{10(cap_{batt})}\right) \qquad len_{min} = \text{ceil}\left(\frac{\text{cpr}_{\text{min}}}{\text{cpr}_{\text{batt}}}\right)$$

Ultimately, our battery storage system will be a matrix consisting of series groupings run in parallel. Within this matrix will be 2 sizes of series groupings, defined above by len_{series} and len_{min}, for the number of maximum and minimum batteries in a parallel grouping respectively. The quantity of these series within our complete battery storage system is further defined by Q_{series}. With these variables defined, we can solve for the exact number of batteries our house requires as follows, and use that value to solve for cost:

$$batt_{Tot} = len_{min}\left(len_{parallel} - Q_{series}\right) + \left(len_{series} \times Q_{series}\right)$$

$$\text{Price} = cost_{batt}\left(batt_{tot}\right)$$

With this, we have successfully generated a model to evaluate a sufficient battery storage system for our house, and also generate its cost. The only remaining step is to plug in specific batteries and their specifications. We used the following table of batteries to complete this process:

Battery	Cost (USD)	Battery Type	CPR	Usable Capacity (KWH)	Weight	Round-Trip Efficiency (%)
Variable	N/A	N/A	cpr_{batt}	cap_{batt}	N/A	N/A
Deka Solar 8GCC2 6V 198	$368	SGLA	0.049 kW	1.18 kWh	68	80-85%
Trojan L-16 -SPRE 6V 415	$492	FLA	0.19 kW	2.5 kWh	118	80-85%
Discover AES 7.4 kWh	$6,478	LFP	6.65 kW	7.4 kWh	192	>95%
Electriq PowerPod 2	$13,000	LFP	7.6 kW	10 kWh	346	96.60%
Tesla Powerwall+	$8,500	NMC	7 kW	13.5 kWh	343.9	90.00%
ENCHARGE-10-1P-NA[19]	$8,955	LFP	3.84 kW	9.7 kWh	346	96%
Fortress Power 48[20]	$3,850	LFP	5.04 kW	5.26 kWh	108	98%
LFPGC2-12175 LiFePO4[21]	$1,099	LFP	1.28 kW	2.2 kWh	38.5	>98%

Table 3c – A collection of the batteries we considered and their statistics. If a source is not cited next to a battery, it came directly from COMAP.

Evaluating the formulas in 3.4-3.6 utilizing the variables defined in table grants us the final result for our standard 1600ft^2 house: To power the house through an entire cloudy day solely off stored battery energy you need 4 Tesla Powerwall+ batteries, totaling a cost of $34000USD. Based on the market-priced 40kWh home backup battery sold by Solar Electric[24], which costs $35,177USD, our value is accurate and representative of current market cost and capacity for solar battery storage systems.

4 – Generalization

Following the generation of our basic model (Addressing Question 1), we must now generalize our model to encompass any home. From our pool of variables, we chose 6 to generalize: *n* (night length), e_L/W_{Light} (Energy use/power draw of lighting), e_{HVAC}/W_{HVAC} (Energy use/power draw of HVAC), and e_{App} (Energy use of appliances). While this list could include more, these variables were simplest to generalize, and thus fit for our timeline.

4.1 – Light

As per assumption 5, lighting will be used during all productive time, which includes some night hours. Because productive night hours pull from the battery array, lighting power will impact our capacity and CPR, and it must be calculated as a function. To accomplish this, we must understand a few factors: how well lit the home is, how much of the home must be lit, the efficiency of our lights, and the impact of people.

Much of this data required research. We found the most efficient LED bulbs on the market can output 116 lumens per watt[1]. Additionally, using consumer guidelines[2], we reasoned that 35lum/ft^2 would be an appropriate level of lighting. This exact light density would vary in a real space, but solving such is beyond our scope. Finally, we arbitrarily chose 150ft^2 as a realistic amount of area 1 person would have lit. With these values, we can create a rough function for light energy usage in kW (W_{light}) with respect to *p*:

$$\mathrm{W}_{light}=\frac{35\dfrac{\text{lum}}{\text{ft}^2}\left(150\text{ft}^2(p)\right)}{116\dfrac{\text{lum}}{\text{W}}}\cdot\frac{1\text{kW}}{1000\text{W}}$$

However, this formula has an issue- as *p* increases, the area that must be lit also increases with no bound, even though the home has a finite *hf* (square footage). Thus, we must create a piecewise function bounded by *hf* as follows:

$$\mathrm{W}_{light}=\begin{cases}\dfrac{35(150p)}{116}\cdot\dfrac{1}{1000},0<p\le\dfrac{hf}{150}\\ \dfrac{35hf}{116}\cdot\dfrac{1}{1000},p>\dfrac{hf}{150}\end{cases}$$

e_L, the energy consumption of light over 24 hours in kWh, is W_{light} multiplied by 24hrs:

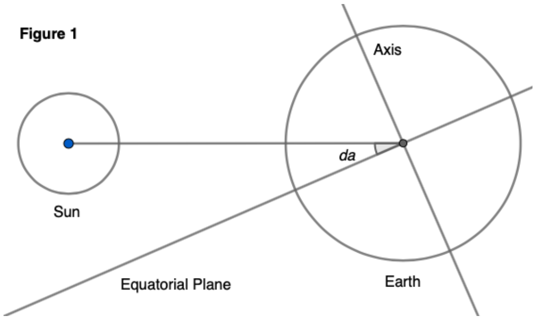
$$e_L=24\text{hrs}\left(\mathrm{W}_{\text{lights}}\right)$$

These functions for light will henceforth be used in our calculations.

4.2 – Night

Due to assumption 2, the length of night (*n*) directly impacts how long our home must work solely off stored energy. In our base model, this length was set, assuming a latitude of 45°. For our full model, however, *n* must be generalized. To achieve this, we must solve for sunrise and sunset times, find the duration of the day, and subtract that value from 24. This process will grant us the length of any night.

According to the NOAA[4], solving for sunrise and sunset times requires 2 distinct variables: The hour angle of the sun at sunrise/set (*ha* in figure 2), and the degradation angle of the Earth (*da* in figure 1). *da* changes slowly throughout the year and *ha* changes based on latitude, however, we only need to consider *ha* as discussed in X.X. It is known that the winter solstice, December 21, (July 21 for the southern hemisphere), has the longest night of any year- when *da* is equal to ± 23.45, the tilt of the Earth's axis with relation to the sun. These are the values we will use for *da*.



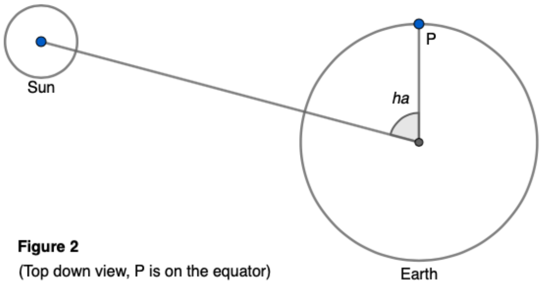


Figure 1 – da *is defined as the angle between the equatorial line and a line drawn from O, the center of the earth, to the center of the sun, or the declination angle.*
Figure 2 – given p is a point on the surface of the earth and O is the center of the earth, ha *is the angle of the line PO, and O to the center of the sun.*
Image credit to Geogebra[23].

With *da* found, we only have to solve for *ha*; this can be done trivially. While it is beyond our abilities to generate an equation for ourselves, the NOAA provides a general equation[3] that we can simplify to meet our needs:

$$ha = \arccos\left(\frac{\cos\left(2\pi\left(\frac{90.833}{360}\right)\right)}{\cos\left(2\pi\left(\frac{lat}{360}\right)\right)\cos\left(2\pi\left(\frac{23.45}{360}\right)\right)} - \tan\left(2\pi\left(\frac{lat}{360}\right)\right)\tan\left(2\pi\left(hemi\,\frac{23.45}{360}\right)\right)\right)$$

Where *hemi* is -1 for the northern hemisphere and 1 for the southern hemisphere.

Using this definition for *ha*, we can now solve sunrise/set times. Hour angle is simply the angle of the sun against a vertical line on the Earth's surface (as in figure 1), meaning an hour represents 15° (360°/24hrs). Thus, the time of *n* (in hours) can be calculated as:

$$n = 24\text{hrs} - 2\left(\frac{\frac{360}{2\pi}(ha)}{15°}\right)\text{hrs}$$

This function for *n* will henceforth be used in our equations.

4.3 – HVAC

Another appliance that factored heavily into our equations was the energy use of HVAC. To make our model more ideal, we decided to account for its usage of power and energy as a function of temperature and square footage. Through preliminary research, we generated the following basic function for energy use:

$$e_{HVAC} = \frac{A\left|(t-c)\right|}{r} \cdot 24\text{hrs}$$

Where A is the surface area of the house, t and c are the indoor and outdoor temperatures respectively, and r is the thermal conductivity of the house. This function calculates the energy consumption of the HVAC system by finding how much heat needs to be replaced in BTUs (British Thermal Units) over a day.

However, different parts of the house have varying values of thermal conductivity, which we considered a large factor in calculating the energy consumption of HVAC. To make our data more accurate, we incorporated these different parts into our model with respect to H_f, the square footage of the house. To do this we inserted the surface area of each part A_x into the equation as A, and the thermal conductivity of said parts r_x into r. For simplicity, we only considered the surface area of the walls, windows, and roof:

$$e_{HVAC} = \frac{A_x\left|(t-c)\right|}{r_x} \cdot 24\text{hrs}$$

With these parts inputted, we can accurately calculate the energy consumption of HVAC by repeating this equation for all parts of the house and adding those equations together.

$$e_{HVAC} = 24\text{hrs}\left(\left|(t-c)\right|\left(\frac{A_{walls}}{r_{walls}} + \frac{A_{win}}{r_{win}} + \frac{A_{roof}}{r_{roof}}\right)\right)$$

Now that we have an equation that finds the heat needed to be replaced, we must convert it into energy consumption. We did this by incorporating the ratings designated by the manufacturer for their HVAC units, which are measured in BTUs per watt. Thus, we divided our equation by these ratings multiplied by 1000 to output a value in kWh:

$$e_{HVAC} = 24\text{hrs}\left(\left| (t - c) \right| \left(\frac{\frac{A_{walls}}{r_{walls}} + \frac{A_{win}}{r_{win}} + \frac{A_{roof}}{r_{roof}}}{R_{heat} \cdot 1000} \right) \right)$$

Where R represents the manufacturer rating. There is a difference in the capability of the HVAC system to heat and cool a house, as shown in the different ratings assigned by the manufacturers denoted as SEER (cooling) and HSPF (heating). To account for this, we used a piecewise function incorporating our previous equation:

$$e_{HVAC} = \begin{cases} 24\text{hrs}\left((t - c) \left(\frac{\frac{A_{walls}}{r_{walls}} + \frac{A_{win}}{r_{win}} + \frac{A_{roof}}{r_{roof}}}{R_{heat} \cdot 1000} \right) \right) & t > c \\ 24\text{hrs}\left((c - t) \left(\frac{\frac{A_{walls}}{r_{walls}} + \frac{A_{win}}{r_{win}} + \frac{A_{roof}}{r_{roof}}}{R_{cool} \cdot 1000} \right) \right) & t < c \end{cases}$$

This piecewise function allows us to differentiate between when we need to heat or cool, based on if the indoor temperature is warmer or colder than the outdoor temperature.

For the sake of our model, we assigned values to these parts based on research regarding the layout of housing, the thermal conductivity of materials used, our designated heater, and the 1600ft^2 house provided by the initial prompt:

$$e_{HVAC} = \begin{cases} 24\text{hrs}\left((t - c) \left(\frac{\frac{1344}{20} + \frac{96}{2} + \frac{1600}{30}}{10 \cdot 1000} \right) \right) & t > c \\ 24\text{hrs}\left((c - t) \left(\frac{\frac{1344}{20} + \frac{96}{2} + \frac{1600}{30}}{20 \cdot 1000} \right) \right) & t < c \end{cases}$$

Our equation at this point determines the energy consumption of a HVAC system running at maximum output. However, it is fair to assume that a HVAC will not always run at maximum output. Because of this, we incorporated a function that calculates the wattage required based on the area of the house using the heater we used as a baseline:

$$W_{HVAC} = \left| \frac{1.642}{h_f} \right|$$

Where 1.642 is the wattage of the heater used in our model.

These functions for HVAC will henceforth be used in our calculations.

4.4 – Appliances

Electrical appliance usage is a significant element of power consumption in a home, creating much of the electricity demand. To create an accurate model, it is a necessity to consider this consumption. Due to the extreme variability between appliance usage in homes, we first estimated the average appliance usage of a four-person family in the United States, the number of occupants whom we assumed would be living in our 1600 square foot home, and then we defined the usage of appliances in homes with a variable occupancy relative to that baseline. As per the design of our model, we need to find the combined power draw of the home's appliances (in kW) and the total amount of energy used throughout a day (in kWh). These values will be represented by the variables W_{App} and e_{App} respectively.

We reasoned that the combined power draw of appliances would be equal to the total wattage of all appliances being run regularly (in kW), divided by two. It was then assumed that no household, especially one supported by solar energy, would have more than one-half of all appliances running at once. Based on our research referenced on pg. 6, our equation for W_{App} for a four-person household is:

$$W_{App} = \left(\frac{1.8\text{kW} + 1.32\text{kW} + 2.8\text{kW} + 4.8\text{kW} + 1.7\text{kW} + 0.106\text{kW}}{2} \right)$$

After the creation of this model, we further reasoned that the power draw of appliances did not scale directly as the number of people in a household did, as the number of appliances in a household does not have a strong relation to the number of inhabitants in the household. Therefore, the maximum power drawn from those appliances would stay the same. This means that the above equation can be plugged directly into our general model.

After tackling the power draw of a home's appliances, we must move on to the total amount of power used by said appliances. Adding together the daily energy consumption of each appliance we researched yields e_{App} for our four-person house. This value is 5.84 kWh. To adapt our definition of e_{App}, we determined that the power is by appliances is proportionate to the number of people in a household. After converting e_{App} into a function of p, our equation becomes:

$$e_{App} = \begin{cases} 5.84\text{kWh}, & 0 < p \leq 4 \\ \dfrac{p}{4} \cdot 5.84\text{kWh}, & p > 4 \end{cases}$$

This equation sets a minimum for the energy consumption of appliances at 5.84 kWh, as even in a household of fewer than four people, there is a minimum usage of appliances, appliances cannot be run a partial number of times. Thus, for any input of p less than four, the output will be equivalent to a four-person home. For inputs of p greater than 4, the base value of

5.84 kWh is multiplied by p divided by 4, the ratio of the inputted household size compared to the standard we used.

4.5 – Implementation

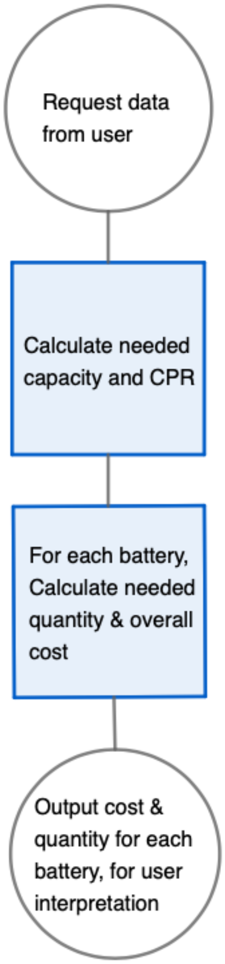
With all our variables generalized, we can commence their implementation into our prior model (see section 3, page 5). Substituting in our variable solutions, we get the general equation:

$$E_n = \frac{n}{24}\left(e_{HVAC} + e_c\right) + \frac{n-8}{24}\left(e_L + e_{App}\right)$$

$$E_d = \frac{24-n}{24}\left(e_{HVAC} + e_L + e_{App} + e_C\right)$$

$$E_{Cloudy} = E_d + 2E_n$$

Where e_C is adapted to not include HVAC.



While this equation is still possible to evaluate manually, due to the quantity of values and batteries we are dealing with (see Table 3c), it is much simpler to automate our solution through Python code. The flow chart for the code can be seen here to the left, but in effect, it is a streamlined calculator which outputs $batt_{tot}$ and price for each of our batteries. The full code can be seen under the appendix on page 22. With our generalized model implemented, we can continue our full analysis and conclusion.

Figure 3 – The 'flow' of our evaluating code.

5 – Conclusion

5.1 – Results

With our model fully generalized, we can commence making our final remarks, beginning with our results. While the exact quantity of batteries need varies wildly depending on inputs (as addressed in 5.2), the chosen battery type stays almost constant, overwhelmingly being the Tesla Powerwall+ (from Table 3c). While this consistency may seem a boring result, it makes sense in the context of the problem. This battery data is based on real life, it is not hypothetical. As such, you would expect the efficacy of available batteries to fluctuate, with one superior contender among the mix. In our case, this contender happened to be Tesla's entry, but as the battery industry develops that will very likely change. Overall, addressing this issue is an outstanding strength of our model: if we choose to add a battery into our consideration in the future, we simply need to add it to a list (in code under appendix, pg.22), and it will be considered.

5.2 – Sensitivity Analysis

The sheer amount of data our model encompasses means some variables will ultimately be buried and have little impact, while others are magnified beyond belief. In this rudimentary analysis, we will examine such variances in importance, the 'sensitivity' of our model.

The "base" inputs for our model are:

- 45° Latitude (*lat*)
- 72°F Internal Temperature (*t*)
- 1600ft^2 Square Footage (H_f)
- 4 People (*p*)

If a change in these inputs is not specifically mentioned, they were held constant.

Varying temperature (*t*), we find that the larger the value of $|t-32|$ and H_f, the more Tesla batteries required. Specifically, a change of ~15° constitutes a change in 1 battery, with a minimum of 3. Considering that HVAC (the appliance impacted by temperature) is a constant, very power-hungry appliance, these drastic changes relative to small differences in temperature make sense.

Varying people (*p*), we find that every increase of 4 brings a change of 1 Tesla battery. Following the logic used in our lighting and appliance generalizations, this is expected. While it is very difficult to find data online which reflects and supports these battery requirements, it would be simple to change the leading coefficients or bounds of our equations to reflect real life. Overall, these changes make sense according to the considerations we made, but they could be fine-tuned should more secure data become available.

Varying latitude (*lat*), we find a somewhat disappointing result: a variance of $\pm 60°$ constitutes a change of only 1 Tesla battery, with a minimum of 4 at the equator. Considering the effort we put into generalizing n, this is unsatisfactory- however, it may have some logic. n only impacts our required capacity after the other calculations have been made. Thus, if our E_{Cloudy} has a low base value, n will have little impact. While still somewhat disappointing, it is reassuring that our model's output with relation to n follows some logic.

Varying square footage (H_f), we find the very nice linear relationship of 1 Tesla battery added per every 1000ft^2. This requires little analysis- H_f impacts both light usage and HVAC usage, and thus it must have a drastic impact. Our model reflects this impact.

Overall, our model could be improved with new thorough data on house power usage, but the values it outputs have a good basis in reality according to our assumptions and equations.

5.3 – Strengths and Weaknesses

Considering our limited timeframe, our model has some very apparent strengths and weaknesses. Among our strengths is robustness/room for expansion, built-in redundancy, a basis in true data. The first strength we believe is quite apparent. Our model considers a fair amount of realistic data to generate its output, and the amount of data it considers can very easily be altered. For example, if we were to make our model more robust by calculating TV energy usage with some variable, all that would be required is to put that equation under productive power in the solution for E_{Cloudy}. This is true for effectively any appliance. Furthermore, our model has built-in redundancy. By accounting for the worst-case scenario, we ensure that our battery system can last through just about anything.

We now must address our assumptions, the basis of most our weaknesses. We had to assume quite a bit- while our base assumptions (see 2.1) were acceptable due to the limits of our knowledge and time, we had to make various little assumptions for coefficients throughout our model, and these were pretty shaky. If we had more time, it would have been beneficial to research more, increasing the strength of our model as a whole. Considering the output of our model itself, we only considered 1 battery at a time, a possible weakness. Instead of rounding up on our equation for total batteries, we could have rounded down and used smaller batteries to meet the remaining needed CPR or capacity, making our battery storage system cheaper. These two weaknesses were the most apparent throughout our paper, but they were certainly not alone. However, considering the short amount of time we had to generate our model, we believe it is nonetheless quite strong.

6 – Supplemental Writings

6.1 – Cement

Cement barrerites are a recently developed technology permitting the implementation of electrical storage within cement, a widely used construction material. While still a very young concept, the batteries show promise, and could eventually be a cornerstone of any city's infrastructure. As such, we decided to make a brief consideration over their application, and discussed the possibility of their induction to our model.

We began our consideration by exploring the inherent advantages and disadvantages of cement batteries. These batteries provide versatility in many use-cases (houses, buildings, sidewalks, etc.), only require modifications to pre-existing and widely used materials, and are easily rechargeable and cost-effective. However, there are some trade-offs, the biggest of which is the low capacity relative to volume (the technology is only able to store 7 Wh/ft^2[7].

We explored the possibility of integrating these batteries into our house, it could be constructed of the batteries themselves, creating a "structural battery". The batteries' usage as structural components for the house means that no extra space needs to be dedicated to batteries, compensating for their poor energy to area ratio. However, several more considerations and developments are yet required to model and compare cement batteries to ones currently available on the market. Even with this incredible space efficiency, the cost or efficiency of the batteries may ultimately be their downfall, making their implementation unjustifiable. For now, traditional batteries are superior due to their greater development, but cement batteries are surely a development to follow.

6.2 – News Article

When you panic over a dead T.V. remote or try to power your latest Christmas gift, what do you need? You need batteries! Batteries are a mainstay in electronics, providing a way for energy to be stored and accessed later. Batteries power many household objects and quality of life items for your daily enjoyment. However, did you know that batteries can be used for so much more? Large, block-shaped batteries are used to power equipment and store energy on a much larger scale, such as in a car or, better yet, to store renewable energy.

Imagine you are living off the grid and your only method to harness electricity is through solar panels. These panels may be sufficient for your electrical needs during the day, but what happens during the nighttime? Solar panels cannot generate energy without sunlight, meaning that you would have to somehow store your energy from the day to last through the night. Enter: Batteries. Solar batteries are rechargeable batteries that are perfectly suited to absorb the energy provided by solar panels, and then provide large amounts of energy for extended periods of time; in this case to power a house.

But how do you know what batteries to use? With so many solar batteries on the market, choosing one can prove to be a tiresome ordeal. We rigorously tested 8 different batteries on the market, making sure to validate the cost and quality of each. How much energy do people use? How long does the night last? These were all questions in our test to pick the best battery. Through our research, we concluded that out of the 8 batteries we tested, the Tesla Powerwall+ was the best choice. At $8,500 per unit, the Tesla Powerwall+ provided the largest amount of storage and efficiency per dollar. It simply out-performs all other batteries we tested at a cheaper cost, making sustainable energy accessible to all.

So how is this applicable to you? Solar energy may seem financially daunting at first, with such a high initial cost, but over time its benefits become more apparent. Taking energy from the grid cost an exorbitant amount of money over time, upwards of $140 a month! With solar panels, once you pay the initial fees you essentially get energy for free. The cost of solar panels and batteries may seem too steep as of now, however, as technology advances so too will solar technology, leading to cheaper prices. Even now people are developing new and affordable ways to obtain and store solar energy.

One example of new technologies is the development of cement batteries by scientist in Sweden. These cement batteries work like normal batteries, storing energy within them through a combination of battery like material embedded into the concrete. Although, as of now, these batteries store very little energy and are not completely developed. However, with its potential in storing energy, rechargeability, and availability, we can expect this technology to become useful and applicable development. In the future, we could utilize these batteries where normal cement would be used such as in cities, roads, buildings, and houses, improving the electrical market and allowing for affordable energy storage for all consumers.

7 – Appendix

Code (referenced on pg.15):

```
import math
Battdict = {
    "0": [10, 7.6, .9660, 13000, "Electriq"],
    "1": [13.5, 7, .90, 8500, "Tesla"],
    "2": [1.18, 0.049, .8, 368, "Deca"],
    "3": [7.4, 6.65, 0.95, 6478, "Discover"],
    "4": [10.1, 3.84, 0.95, 8955, "ENCHARGE-10-1P-NA"],
    "5": [5.374, 5.04, 0.98, 3850, "Fortress Power 48"],
    "6": [2.24, 1.28, 0.98, 1099, "LFPGC2-12175 LiFePO4"]
}
'''Dictionary of battery data stored in the following format:
    [Capacity, CPR, Efficiency, Cost, Name]'''


'''Solving for night duration'''
def dtor(x):
    '''Degrees to Radians'''
    rad = (math.pi)*(x/180)
    return rad
lat = math.floor(int(input("What is your latitude? ")))
if lat > 0:
    hemi = 1
else:
    hemi = -1
    # changing solstice based on hemisphere (through dec angle)
ha = math.acos(math.cos(dtor(90.833))/math.cos(dtor(lat))*math.cos(dtor(hemi*-23.45))-math.tan(dtor(lat))*math.tan(dtor(hemi*-23.45)))
degha = ha*(180/math.pi)
```

```
n = 24 - 2*(degha/15)

'''HVAC energy based on internal temperature'''
t = int(input("What temperature do you keep the house at (F)? "))
c = 32 # outdoor temp
sqft = int(input("What is the square footage of the house? "))
Awin = 96
Awall = math.sqrt(sqft)*9 - 96
if t > c:
    eneHVAC = 24*(((t-c)*(Awall/20+Awin/2+sqft/30))/10000)
else:
    eneHVAC = 24*(((c-t)*(Awall/20+Awin/2+sqft/30))/20000)
Whvac = 1.642/sqft

'''Lighting Energy w/ respect to people & sqft'''
p = int(input("How many people live in the home? "))
if 0 < p <= sqft/200-1:
    Wlight = 35*(200*p)/116000
    eneL = 24*Wlight
else:
    Wlight = 35*(sqft)/116000
    eneL = 24*Wlight

'''Appliance Energy w/ respect to people'''
Wapp = 5.84
if 0 < p <= 4:
    eneApp = Wapp
else:
    eneApp = (p/4)*Wapp
```

```
'''Variable bank'''
constantP = eneHVAC + 11.606 # constant power per day, in kWh
productiveP = eneL + eneApp # productive power per day, in kWh
Wconst = Whvac + 4.5 + 0.61

En = n/24*(constantP)+(n-8)/24*(productiveP)
Ed = (24-n)/24*(constantP + productiveP)
Ecloud = Ed + (2*En)
# capacity equations
cpr = (Wapp+Wconst)/2+Wlight
mincpr = Wconst+Wlight
# Solving for needed CPR using equation with set calculated values
Variables = [Ecloud, cpr]
# Data list for easy access

'''Final calculations of h, k, and j'''
for batt in range(0, len(Battdict)):
    k = math.ceil(Variables[0]/((Battdict["{}".format(batt)][2])*(Battdict["{}".format(batt)][0])))
    h = math.ceil(Variables[1]/Battdict["{}".format(batt)][1])
    j = math.ceil(Ecloud/(8*(Battdict["{}".format(batt)][2]*Battdict["{}".format(batt)][0])))
    g = math.ceil(mincpr/Battdict["{}".format(batt)][1])
    totalbatt = g*(k-j) + (h*j)
    totalcost = totalbatt * Battdict["{}".format(batt)][3]
    costlist.append(totalcost)
    print("You will need {0} {2} batteries, at a cost of $ {1}".format(totalbatt, totalcost,
Battdict["{}".format(batt)][4]))
    batt =+ 1
```

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